ERROR PATTERNS, CONCEPTUAL CHANGE AND ACCELERATED FORGETTING: ANOTHER DIMENSION TO THE JIGSAW OF EFFECTIVE CONCEPTUAL MEDIATION IN MATHEMATICS.

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Traditionally, students' mathematics errors and misconceptions were viewed from a negative perspective, taken as indicative of the absence of knowledge/meaning. Constructivist theory offers a more positive perspective, suggesting that errors are an individual's current interpretation of a mathematical situation and thus are indicative of knowledge. Error pattern research has prompted new approaches to intervention, with errors/misconceptions increasingly being used as the beginning point for intervention. The success of such approaches has been mixed with error recidivism being a common occurrence. A further dimension to this field is offered by Conceptual Mediation (CM) (Lyndon, 1995). The theoretical background of CM states that accelerated forgetting of new material occurs if it conflicts with pre-existing knowledge. Errors/misconceptions therefore are retained even in light of rational argument. In this paper, error pattern research and conceptual change programs are briefly summarised, followed by a discussion of the psychological basis of CM.

Errors and Error Pattern Research

The Contribution of Error Pattern Research

The study of students' mathematical errors/misconceptions has significantly influenced the field of mathematics assessment and intervention, providing alternative perspectives as to what error patterns indicate (e.g., Ashlock, 1994; Ashlock, Johnson, Wilson & Jones, 1983). Traditionally, students who made errors in their work were regarded as suffering from some type of learning disability (e.g., Kephart, 1960), and that they made errors because they lacked knowledge of "correct" algorithms. Such a deficit model of error production suggested that the student exhibiting the error had learned nothing as a result of the initial teaching effort.

Error pattern research has revealed that, contrary to the belief that all errors are random and careless, they occur regularly and consistently (Brumfield & Moore, 1985; Cox; 1975). Consistency in production of errors tends to negate such a view that errors are indicative of a lack of knowledge. According to Ashlock (1994), the fact that errors can be systematic over certain mathematical computations indicates that they are habitual, automatic responses to specific stimuli. In contrast to random, careless errors, habitual errors are not self-detected nor self-corrected; they are conceptual and learned. The implication of errors as conceptual and learned knowledge provides an alternative perspective on what errors indicate about a student's mathematics knowledge. Errors are thus indicative of the presence rather than the absence of knowledge. The notion of mathematics learning disabilities suggests a difficulty in acquiring knowledge. Consistency in errors indicates that the student is, in fact, capable of learning. From this perspective, what a student has learned is merely an incorrect way of doing things. The student has somehow acquired a learned disability rather than a learning disability (Ashlock, 1994).

Categories of students' (and adults) patterns of mathematical error have been well documented over many mathematical domains. For example, Ashlock (1994) provided a comprehensive historical summary of error pattern research, focussing particularly on identification of error patterns in computation. Some relatively recent studies have reported on evidence of consistency in students' (and adults) errors in mathematical skill calculation and conceptual understanding in other mathematical topics, such as Year 8 students' understanding of parallel lines (Mansfield & Happs, 1992), Year 5 students' understanding of ratio and proportion (Fong, 1995), Year 10 student's understanding of circle geometry (Borassi, 1994), high school students' skill in factoring polynomials (Rauff, 1994), secondary school teachers' concepts of group theory (Dubinsky, Dautermann, Leron, & Zazkis, 1994).

The value of error pattern research can be seen to operate on at least three levels. Primarily, the accuracy of the diagnosis will enable specific intervention strategies and activities to be developed, with a greater chance of successfully helping the student overcome the learning difficulty and progress towards mathematical achievement. At a second level, error pattern research has pedagogic implications. If it is known the various errors students develop in relation to particular mathematical topics, teachers can develop programs of instruction in an effort to possibly prevent the development of such errors (Maurer, 1987; Stefanich & Rokusek, 1992). The creation of appropriately rich learning environments can thus be created from an informed position with greater teacher awareness of possible student misconceptions that may arise from the teaching experience. The study of systematic errors benefits teaching in that sources can be determined and learning environments developed that inhibit errors (Behr & Harel, 1990). At a third level, error pattern research has implications for teacher training programs. For example, Thipkong and Davis (1991) alerted educators to the influence of teacher errors and misconceptions upon their teaching, and thus on student learning. In their research, they identified preservice teachers' misconceptions in interpreting and applying decimals, noting that the misconception "multiplication makes bigger, division makes smaller" extremely prevalent. They suggested that if teachers are aware of their own errors and misconceptions in particular mathematical topics, great care will need to be taken so that such errors and misconceptions are not transferred to learners. This research serves to thus inform mathematics teacher training programs.

<u>Errors as Constructed Knowledge</u>

Constructivist theories of learning state that knowledge is actively constructed by the individual. Of constructivism, Confrey (1990a) stated, "constructivism can be described as essentially a theory about the limits of human knowledge, a belief that all knowledge is necessarily a product of our own cognitive acts" (p. 108). Interpretation of mathematical errors and misconceptions as knowledge, then, is consistent with constructivist learning theory. Errors are personal constructions in the mind of the individual, and thus are meaningful, and make sense, to the individual (Confrey, 1990b; Rauff, 1994). They are an individual's interpretation of a mathematical situation at the time. Confrey (1990a) stated that the work of Piaget served to highlight the concept of knowledge as mental constructions, as encapsulated in the following paragraph:

> ...a child may see a mathematical or scientific idea in quite a different way than it is viewed by an adult who is expert or experienced in working with the idea. These differences are not simply reducible to missing pieces or absent techniques or methods; children's ideas also possess a different form of argument, are built from different materials, and are based on different experiences. Their ideas can be qualitatively different, which can sometimes mean that they make sense only within the limited framework experienced by the child and can sometimes mean they are genuinely alternative. To the child, they may be wonderfully viable and pleasing. (p. 108-109)

Confrey's statements provide a clear picture of active learners operating on and interpreting mathematical and scientific situations within their own mental framework. Further, Borassi (1994) provided a comprehensive description of the growth of mathematical knowledge at a societal level to reflect mathematical knowledge growth at an individual level in terms of constructivism. Citing the philosophical contributions of explanatory theories of mathematical knowledge, including Dewey, who suggested that knowledge is a process of inquiry motivated by doubt; Kuhn, who described knowledge as oscillation between normal science and scientific revolution, where unacceptable results and unsolvable problems leads to new perspectives; and Lakatos, who stated that mathematical knowledge occurs through a dialectic process of proofs and refutations; Borassi stated that mathematical knowledge can be seen to be a constructed body of knowledge, changing and evolving over time. In light of this philosophical analysis of the construction of mathematical knowledge through history, Borassi likens the growth of mathematical knowledge in students. He described a view of learning "as a generative process of meaning making that is personally constructed, informed by the context and purposes of the learning activity itself, and enhanced by social interactions" (p. 167). In terms of errors and misconceptions, Borassi's discussion shows that, at certain times in history, mathematical knowledge was erroneous, but was the socially constructed mathematical knowledge of the time. Mathematical errors and misconceptions, therefore, can be regarded as constructed knowledge. As Confrey (1990a) argued, "students are always constructing an understanding for their experiences...Students' misconceptions, alternative conceptions and prior knowledge provide evidence of this constructive activity" (p. 112). In a similar vein, Rauff (1994) described students' mathematical knowledge as constructed over time; students' mathematical knowledge is based on their beliefs, and errors and misconceptions stem from their beliefs. According to Rauff, errors are logically based from within the student's "belief-set" and are thus meaningful and logical to the owner.

The Development of Consistent Errors

In relation to the development of consistent patterns of error, Resnick and others (Nesher, Leonard, Magone, Omanson, & Peled, 1989) provided a description of how students develop patterns of error in computation, which can be seen to fit within a constructivist framework. Resnick et al. stated that "errors derive from students' attempts to integrate new material that they are taught with already established knowledge" (p. 25). In further explaining this process, Resnick et al. suggested that within the mathematics classroom, teachers provide various examples of mathematical procedures for students to learn and practise, but that within this classroom situation, teachers can only provide a certain number of examples. When students are faced with computation exercises that have not been explained by the teacher, students must decide for themselves how to proceed. According to Resnick et al., as a result of "making these inferences and interpretations, children are very likely to make at least temporary errors. Errorful rules are a natural result of children's efforts to interpret what they are told and go beyond the cases actually presented...[Thus] errorful rules are active constructions" (p. 25). Errorful rules therefore can be seen as interpretations of the child's view of the mathematical situation at the time.

Brown & Van Lehn (1982) also provided an explanation for the development of error patterns in which errors can be regarded as active constructions. They described the development of error patterns in terms of computer language with students' errors labelled as "bugs", and the process through which these bugs develop as "Repair Theory". Repair Theory states that when learners are confronted with tasks on which they are unsure of how to perform (on which they have become "stuck"), they use a simple "repair" tactic which enables them to produce a solution and become "unstuck". In this way, repairs occur as a result of learners' choosing alternative solution paths in order to produce answers. Repair Theory also states that if the repair is erroneous and is left unchecked, the incorrect repair, through repetition and practise, will become a habit produced in response to appropriate stimuli. The repair thus becomes a consistent error; that is, a "buggy" solution. Encompassed within this theory is "bug migration" to explain the fact that some students take several alternative solution paths in response to the one stimulus, hence switching between bugs. Succinctly, Repair Theory is an explanatory theory for the development and consistency of erroneous algorithmic procedures (buggy solutions), and the existence of several incorrect procedures for the same stimulus (bug migrations). [Bug migration does, however have implications for accurate diagnosis of consistency in errors. Some procedures yield correct solutions, thus confirming the legitimacy of the buggy procedure in the mind of the student, and hence making remediation of that error pattern all the more difficult (Ashlock, 1994)].

Overcoming Mathematical Errors and Misconceptions

Reteaching Approaches

For the remediation of systematic errors, suggested approaches predominantly appear to be good reteaching programs which emphasise the close linkage of the symbolic representation with the concrete/pictorial representation in order to promote conceptual understanding (e.g., Ashlock, 1994; Booker, Irons & Jones, 1980; Resnick, 1982; J. W. Wilson, 1976). In such approaches it appears that the teacher acknowledges students' error patterns, but they are not overtly referred to in the intervention situation. This reteaching approach has not always resulted in sustained conceptual change and eradication of the error. For example, Resnick (1982) in focusing on students' computational procedures for subtraction algorithms found that, as a result of intervention, the students in the experimental group, with intensive instruction using concrete materials and place-value games, performed only marginally better than students in the control group. Of this study, Resnick (1992) stated:

> Despite the intensive personal instruction, only half the children taught learnt the underlying semantics well enough to construct an explanation of why the algorithm worked and what the marks represented. More surprisingly, even children who did give evidence of good understanding of the semantics often reverted to their buggy calculation procedures once the instructional sessions were over. (p. 394)

Similarly, Connell and Peck (1993) found that, despite the use of concrete materials, students' prior knowledge interfered with their ability to perform computations correctly; the old, erroneous procedures continually resurfaced. Other studies specifically designed to help children overcome errors in computation have reported that students' old error patterns reemerge despite the intensity of the remedial activities (e.g., Bourke, 1980; Resnick, 1982; Wells, 1982; N. Wilson, 1982). It is acknowledged however, that particular studies have reported that the use of "good teaching" strategies will help students overcome error patterns in computation (e.g., Stefanich & Rokusek, 1992), and that students errors may naturally correct over time (Hennessy, 1993).

Current Trends in Mathematics Intervention

Alternative programs have been described in the literature which appear to actively use students' error patterns/misconceptions as a focal point for intervention and conceptual change. Such programs include error pattern analysis and intervention (e.g., Ashlock, 1994; Gable, Enright & Hendrickson, 1991); cognitive conflict and conflict teaching (Bell, 1986-87); using errors as springboards for enquiry (Borassi, 1994); belief-based teaching (Rauff, 1994); teaching by analogy (e.g., Tirosh, 1990). In the following section, a summary of each of these alternatives to traditional mathematics intervention programs is presented.

<u>Correcting error patterns in computation.</u> A comprehensive instructional program for overcoming students' error patterns in computation has been developed by Ashlock (1994) where specific activities are suggested to help students overcome particular errors. For

example, to assist the student who responds to written addition exercises in the following manner:

26	60	74
+ 3	+ 24	+ 5
11	84	16

(taken from Ashlock, 1994, p. 133)

Ashlock suggested the provision of place value identification games, using base ten blocks or paddle pop sticks (for bundling into tens and ones) to represent each addend in the exercise, and the drawing in of place columns to display the tens and ones in each of the numbers. The suggested activities aimed to build the student's understanding of place value and to demonstrate the illegitimacy of his/her solution process. The focus can be seen to be on the particular computation in which the error pattern surfaced.

A much more prescriptive approach to correction of error patterns has been offered by Gable, Enright, & Hendrickson (1991). They described a three-phase model of analysis, intervention, and evaluation. In their approach, the first phase involves determining the consistency of the error and includes interviewing the student. In the second phase intervention begins, and involves the three stages of (i) demonstration of the correct algorithm, (ii) selection of "the error groups and appropriate corrective strategy" (p. 7), and (iii) practise of the new algorithm. The appropriate corrective strategy is determined through categorising the nature of the error as either conceptually-oriented or structurally-oriented. As Enright et al. stated, "conceptually-oriented error patterns, such as regrouping errors and place value problems, require a manipulative, hands-on corrective strategy. In contrast, error patterns such as process subtraction, placement, and attention to sign can be corrected using graphically oriented strategies including the use of flowcharts or colour coding to structure the work page" (p. 7). In this approach, phase two is characterised by considerable practise of the new/correct computational procedure. In phase three, the evaluative phase, transfer of the skill to the regular classroom is evaluated. This phase has two stages: (i) the impact of the new skill on student performance is evaluated with the student, and (ii) the practise and maintenance of the skill is continued in the classroom context. The three phase model can be seen to be cyclic, and as Enright et al. claim, can be used in the regular classroom as it integrates within a curriculum-based assessment and instruction mathematics program.

Cognitive conflict and conflict teaching. Cognitive conflict models of instruction are based on the premise that prior inappropriate knowledge serves as a barrier to knowledge growth and development, and that this inappropriate knowledge must be confronted (Bell, 1986-87). In such teaching situations, the environment is structured so that students' misconceptions will surface as students work on mathematical tasks deliberately developed by the teacher for that purpose. Through discussion in group situations with peers and others, students' misconceptions are brought into the open. Through discussion, the intention is that students will see the impoverishedness of their understandings, and thus conceptual change will occur.

Conflict teaching appears to be based on acknowledging the power of prior learning. However, such an approach does not always result in sustained conceptual change occurring, as students' misconceptions are often in evidence after such conflict exercises (Bell, Swan, Onslow, Pratt & Purdy, 1985; Tirosh & Graeber, 1990). Even though students can see the limitations of their own conceptualisation within a particular topic, they can develop and hold appropriate concepts without giving up their prior, inappropriate concepts. The prior-held misconception continues to interfere with understanding and forward learning. This phenomenon has been described as due to knowledge compartmentalisation, where two conflicting ideas are held as separate entities in the mind (Posner, Strike, Hewson & Gertzog, 1982; Vinner, 1990). Posner et al. (1982) suggested that compartmentalisation is a learner's mechanism for avoiding cognitive conflict and conceptual change. This perspective suggests that human learners actively, though unconsciously, resist cognitive conflict. As Tirosh (1990) stated, "in cognitive psychology, human beings' desire to eliminate conscious inconsistencies in their thinking is regarded as a basic cognitive need" (p. 111).

Together with the fact that conflict teaching does not always lead to sustained conceptual change, is the fact that conflict teaching requires learners to openly display the extent of their inappropriate knowledge so that critical peer review and analysis can occur. This calls into question the effect of such an approach on students self-esteem and confidence. As Tirosh (1990) cautioned, "the conflict teaching approach includes a stage in which a student realises that something in his or her way of thinking is "wrong". In certain cases, this realisation may actually be detrimental to a student's confidence or self-esteem" (p. 123).

<u>Errors as springboards for inquiry.</u> Borassi (1985) stated that in the field of diagnosis and remediation, errors are regarded in a negative fashion as "signals that something has gone wrong in the learning process, and consequently remediation is needed" (p. 1). He suggested that errors should be viewed from a more positive perspective as the means to promote students' thinking about mathematics and thus build mathematical understanding. Motivated by the belief that errors can be used to develop students' deeper understanding of mathematics, Borassi (1994) conducted a teaching experiment focusing primarily on using students' errors for student inquiry. In his study, he collected students' written definitions of a circle. He then presented these definitions to other students, after asking them to write their own. The students were required to analyse each definition and compare and contrast it with their own definition, thus modifying, rejecting, arguing for, justifying, and so on, certain definitions of a circle. The teacher's role was to assist the inquiry process, prompting students to explain clearly their statements, probing their knowledge of circles, and using this to continue the growth of the definition along appropriate lines. According to Borassi, the strategy of using "errors as springboards for inquiry" appeared to not only help students change and modify their current conceptions of the mathematical topic under study, but also engaged them in "genuine problem solving, mathematical explorations, mathematical communication, initiative and ownership in learning mathematics, constructive doubt and conflict, and the need to monitor and justify their mathematical activity, as well as more humanistic and exciting aspects of mathematics" (p. 199). Borassi also reported that the students' learning of mathematical content was also increased as a result of the teaching experiment, as well as the affective domain of the students, with students feeling more positive about the study of mathematics, and their own ability to continue with the study of mathematics.

<u>Belief-based teaching.</u> Rauff (1994) suggested that a process of "belief-based teaching" can help students overcome inappropriate mathematical procedures, and described this in terms of students' erroneous solutions for factoring polynomials. In a process similar to Borassi (1994), Rauff suggested that, to overcome students' errors/misconceptions, beliefs about particular mathematical procedures must be determined and the teacher's role is to assist the integration of the appropriate mathematical procedures within the student's belief set. The theoretical stance which underpins his method is that, errors and misconceptions are a "student's belief set" (p. 425), and the development of errors are logical outcomes of the belief set. According to Rauff, "the mathematics teacher who views errors in this way must discern the nature of the student's model and then attempt to modify it appropriately so that the student can work from a mathematically sound belief set"(p. 422).

In his study, Rauff reported on the relative ease with which some students modified their beliefs, and the difficulty of this process experienced by other students which, according to Rauff, was dependent upon the nature of the student's current belief state. For example, one student used a particular strategy to factorise polynomials, which only worked in certain cases. The student was shown another strategy, but continued to use her own strategy first. Over time, the student came to realise that her own strategy was no longer efficient for all cases, but used it for the cases in which it yielded the correct solution. The student's initial belief set was expanded to include the new strategy, as well as her own. According to Rauff, this inclusion of the new strategy was because it "did not entail the removal of any other beliefs about factoring" (p. 424). Rauff summarised belief-based teaching in the following statement:

> The focus of this approach into teaching and learning is student belief. An instructor using this approach to teaching factoring begins with asking the student to tell him or her what they think about factoring. The instructor then analyses their "buggy" factorisings in light of their beliefs. The students are next shown how their beliefs produce non-equivalent expressions. Finally, the students modify their beliefs appropriately (p. 425).

Teaching by analogy. Teaching by analogy is a teaching approach for building students' conceptual knowledge which has been suggested as an approach for assisting students overcome misconceptions (Tirosh, 1990). The basis of teaching by analogy is that the student's prior knowledge is linked, through analogy, to knowledge being presented by the teacher. The teaching by analogy approach can serve as a means for helping students solve analogous tasks, of helping students develop understanding through linking to analogous situations, of guiding teaching to link to analogous experiences, thus taking what is known and linking to what is new. In dealing with misconceptions, teaching by analogy involves presenting students with tasks that they have previously solved correctly, which are analogous to the tasks the student solved incorrectly. The intention is that the student will see the two tasks as analogous, and revise the approach taken for solution. Thus, students revisit a correctly performed task (the anchoring task) in order to change their approach to solving a task on which they initially experienced difficulty (the target task). This approach poses challenges for teachers, because as Tirosh (1990) stated, the teacher must "find a suitable anchor task, and construct a step, or series of steps, from the anchor task to the target task that convinces the student of the validity of the analogy" (p. 123).

Conceptual Mediation

Overview

The Conceptual Mediation program (Lyndon, 1995) provides a further dimension to error patterns and intervention research by offering a psychological perspective on the development of errors and misconceptions and a reason as to why they are difficult to eliminate. The fundamental principle of Conceptual Mediation is the psychological concept of proactive inhibition (PI) (Lyndon, 1995; 1989). According to Lyndon (1995), proactive inhibition is responsible for the recurrent appearance of error patterns and misconceptions despite intensive intervention programs. To overcome the effects of PI, Lyndon states, is to enter into a process of active "conceptual mediation" (hence the title of the approach) in the sense that mediation means "to stand between, to mediate between to reach agreement". In this program, assisting students overcome their errors and misconceptions is a process of actively and overtly mediating between the student's current knowledge state and the new material being presented to the student. The Conceptual Mediation program suggests strategies to assist this process which can be used indirectly by the teacher, or directly by the students themselves. The theoretical basis of Conceptual Mediation and how it relates to the field of error patterns and mathematical intervention research is discussed in the following sections.

Prior Knowledge and Proactive Inhibition

Proactive inhibition is an information protection mechanism, which "is produced by conflicting associations that are learned prior to learning of the task to be recalled" (Underwood, 1966, p. 564). Underwood suggested that, when a person is asked to give a response to a stimulus that differs from the response the person usually gives, the brain can only do so with great difficulty. Underwood provided the following example to demonstrate proactive inhibition in practise:

> If we are told that: 2 x 2 now is 11
> 8 - 4 now is 1
> 3 + 3 now is 27
>
> we can imagine the difficulty we would have in remembering and applying this new information. Interference, indeed frustration might well occur. (p. 516)

Further, Baddeley (1990) stated that proactive interference (inhibition) occurs when "new learning is disrupted by old habits" (p. 40). Baddeley provided the following as an example of proactive inhibition: "Being taught that C means "caldo" which means hot, but none the less 'forgetting' and turning the wrong tap would be an instance of proactive interference" (p. 40). Similarly, of proactive inhibition, Houston (1991) stated:

> Proactive inhibition is not a theory or an explanation. It is a fact, an important one. It refers to the enormous amount of forgetting that can be attributed to the interfering effects of prior learning. The more we learn, or store, the more susceptible we are to this type of interference. (p. 235)

Proactive inhibition then, as a mechanism for protecting knowledge, is activated when new learning conflicts with prior learning. In situations where prior learning conflicts with current learning, old learning will interfere with recall of the new learning. The need for such a mechanism is apparent, as it can be seen that without such an inbuilt knowledge protection system, the human mind would be in a constant state of confusion; a person's knowledge base would be changing continually in the face of new incoming information. It can also be seen that the existence of a knowledge protection system is a two-edged sword, with all prior knowledge, correct or otherwise, being protected from change. The implications of proactive inhibition for intervention in mathematics are immense. Remediation of learning difficulties in mathematics typically requires students to change their response to a particular stimulus, be it an automatic response to a number fact, a completion of an algorithmic procedure, or a conceptualisation of a mathematical topic. The teacher is providing the same stimulus, but is requiring the student to give a new response that differs to the way the student responded previously to that stimulus. In terms of proactive inhibition, the enormity of such a request is realised, and is exemplified by the examples described above.

As previously stated, constructivist views of learning state that errors/misconceptions are knowledge (e.g. Borassi, 1994; Confrey, 1990a; 1990b; Rauff, 1994). Errors/misconceptions are thus indicative of the presence, not the absence, of knowledge. In terms of diagnosis and intervention, the problem is dealing with knowledge, rather than providing learning experiences to "fill-the-gaps" or "link" knowledge as would be wont in an "absence of knowledge" perspective. Acknowledging the mechanism of PI as a part of the human mind, PI can be seen to serve as protection of errors/misconceptions from change. The role of PI is simply to prevent the cognitive conflict. As Tirosh (1990) suggested, avoidance of mental turmoil is the natural tendency of the human mind.

In view of PI as merely a knowledge protection mechanism, it can be seen that PI cannot determine appropriate knowledge from inappropriate knowledge, therefore all knowledge will be protected by PI. Psychological research studies have shown that it is initially learned knowledge which is more powerfully retained in memory over subsequent learning (e.g. refer to Baddeley, 1990; Eysenck, 1977). The mathematics remediation literature has repeatedly stated that once acquired, students' errors, misconceptions and alternative conceptions are extremely difficult to overcome (e.g., Confrey, 1990a; 1990b; Graeber & Baker, 1991; Fischbein, 1987), thus the need for carefully structured, planned and organised initial instruction is of paramount importance (Connell & Peck, 1993). Acknowledging the influence of PI within the intervention situation provides an explanatory theory for the persistence of errors/misconceptions.

To overcome the inhibitory influence of PI over knowledge change, a specific strategy is an integral element of Conceptual Mediation. The strategy is called Old Way/New Way (O/N). The essence of the O/N procedure is upon bringing the learner's "old way" to a conscious level and exchanging it for a "new way" by means of discrimination learning, followed by practise with the correct "new way". There are four steps to O/N, beginning with reactivation of the error memory, where the error/misconception is recalled, then labelling and offering an alternative, where the error/misconception is labelled the "old way", and a "new way" is shown. In the third step, discrimination, the difference between the "old way" and the "new way" is discriminated a total of five times. In the fourth step, generalisation, the new way is generalised and practised in various situations. The O/N method has been described in detail elsewhere (see Lyndon, 1989).

Analysis of the steps in O/N reveals that the student is required to repeat the "old way" a total of five times. Such an approach is contrary to a perception that reactivation of the error pattern will only serve to strengthen that error pattern. This perception is evident in the words of Gagne (1983) who stated that, "to make students fully aware of the nature of their incorrect rules before going on to teach correct ones...seems to me...is very likely a waste of time" (p. 15). Gagne proposed that to overcome errors is to aim for "extinction" (in psychological terms) of that error, as suggested by the following comment: "The effects of incorrect rules of computation, as exhibited in faulty performance, can most readily be overcome by deliberate teaching of correct rules...This means that teachers would best ignore the incorrect performances and set about as directly as possible teaching the rules for correct ones" (p. 15). In the context of the O/N theory, the error is habitual, and more practise will not serve to make it any harder to eliminate.

The O/N procedure shares similarities with other procedures for dealing with errors/misconceptions, as described in previous sections, particularly those presented by Borassi (1994), Gable, Enright and Hendrickson (1991), and Rauff (1994) but its method is more prescriptive. The key element in the O/N strategy is the active and overt discrimination of differences between the student's knowledge and the mathematical knowledge presented by the teacher. O/N appears to be a direct means to deal with personal knowledge or an individual's belief system (e.g., Confrey, 1990a; Rauff, 1994) protected from change by PI.

Conceptual Mediation and Metacognition

When using O/N for the purpose of overcoming systematic computational errors, O/N can be regarded as a metacognitive strategy applied to a particular cognitive strategy; it is cognitive in that it is task specific, but simultaneously, it is metacognitive as it is a highly generalisable strategy applicable to a wide range of situations. O/N can be presented to students in all three training modes suggested by Brown and Palinscar (1982): blind, informed and self-regulatory. In blind training, the O/N strategy would be used with student errors, but no reason for the application of such a strategy would be given. In informed training, O/N would be used with student errors, and students would be encouraged to reflect on the potential use of the strategy in other domains with other areas of difficulty. In self-regulatory training, the theoretical basis of O/N would be shared with students to provide a clear rationale for the value of such a strategy to aid their own learning. The Conceptual Mediation program is a self-regulatory metacognitive training program in which O/N is presented as a key “remedial” strategy. The program takes the form of a communication with students, sharing with them ideas on what is empirically known about attention, memory, and learning, or more specifically, remembering and forgetting. Because the program discussed basic psychological concepts and principles, it takes the form of a “psychology for children” program.

Recognition and recall memory. The first component of the metacognitive training within CMP focuses on remembering. It is generally accepted that the process of remembering is a result of two memory systems: recognition memory and recall memory. The difference between recognition memory and recall memory can be made explicit by examining the tasks performed by subjects in psychological experiments. In typical recall and recognition tasks, subjects are required to learn and remember lists of words. In recall tasks, subjects are required to remember as many studied words as possible after a given time. In recognition tasks, the word lists often comprise paired lists of words; one word invoking the memory of its pair. Subjects essentially have to be able to recognise whether certain words have been presented to them in the given context (Houston, 1991).

In the metacognitive training component of CM program, the terms recognition and recall memory presented to the students, and differentiated. Recognition memory is described as being externally activated and automatic which is prompted by some sensory input. Recall memory is different, as it is the memory that is utilised to remember something that is not present. the key difference between recognition and recall memory is that recall memory is a self-initiated event which operates at either the automatic level or effortful level. That is, a memory can be retrieved automatically, or it will require a certain amount of effort for retrieval. It is suggested to students that to take control of the remembering process is to store information in automatic recall memory where retrieval is self-initiated, rather than externally stimulated. In order to do this, and thus take control of the remembering process, practise is essential. Efficient practise strategies are an essential component for overcoming academic learning difficulties (Derry, 1990).

Natural and accelerated forgetting. The second component of the metacognitive training within the CM program focuses on forgetting. Two types of forgetting are discussed with students. It is suggested to students that forgetting can be either natural or accelerated. Natural forgetting occurs over time, as skills/knowledge learnt are not practised. As suggested by Anderson (1985), a skill that is not practised becomes victim to the process of natural forgetting. In contrast, accelerated forgetting is described to students as very rapid forgetting. In the metactognitive training program the term proactive inhibition is introduced to students as an information protection mechanism, and O/N is demonstrated as a strategy for overcoming PI and taking control of accelerated forgetting. The notion of accelerated forgetting is presented to students as a natural brain process; a process which occurs when learning a new way of doing something conflicts with an already learned procedure for doing the same thing.

In summary, the key points of the metacognitive training component within the CM program, which form the basis of the communication with students, are as follows:

1. Sometimes learning seems easy and sometimes it seems hard. Learning seems hard because it is paying attention, remembering and understanding that we find hard.

2. When we pay attention to particular things, we learn. We choose the particular things to which we pay attention. Paying attention is hard because it requires effort.

3. We have two memory systems: recognition and recall. Recognition memory happens naturally without effort. On the other hand, recall memory is naturally effortful. Recall memory can be either automatic or effortful. Recall memory only becomes automatic through practise.

4. We have two forgetting processes: natural and accelerated. We can take control of natural forgetting through use of efficient practise strategies. We can take control of accelerated forgetting by using the Old Way/New Way strategy.

Central to CM program is the notion that the brain is designed to forget. It is this key phrase: the brain is designed to forget which provides a rationale for the importance of teaching students how to remember. The purpose of CM program is to inform students on how their own brain works so that they can take control of their own learning. Throughout the program, the continual emphasis is on the fact that learning is a result of effort, and the effort must come from the individual. Making students aware of the process of remembering, learning and forgetting may be part way to answering Norman's (1980) statement:

> It is strange that we expect students to learn yet seldom teach them about learning. We expect students to solve problems yet seldom teach them about problem solving. And, similarly, we sometimes require students to remember a considerable body of material yet seldom teach them the art of memory. (p. 97)

Concluding Comments

The Old Way/New Way methodology offers a specific strategy for dealing with the protective influence of PI over knowledge change and growth. Conceptual Mediation offers a metacognitive training component to help learners take control of their own learning. Superficially, O/N may appear to be the means of replacing one mode of behaviour with another; of replacing a habit with a habit. Indeed, O/N was originally developed for use in overcoming habitual behaviours at the "rote" end of the scale, such as spelling errors, letter reversals, body mannerisms. Several studies have been conducted using O/N and CM within intervention programs in mathematics, and whole class mathematics teaching, including upper-primary students' computational procedures in subtraction (Baxter & Dole, 1991; Dole, 1993), post-compulsory students' mathematical computations in basic mathematics courses (Dole, 1995), junior-secondary students' understanding of percent increase (Dole, 1999). In these studies, it was found that O/N was superior in correcting students' systematic errors in computation and in promoting students' confidence and self-esteem; it was efficient in terms of teacher time and effort, and students also appeared to readily make links between computational and conceptual knowledge. Other research has shown O/N to be successful in changing students' alternate Science conceptions in whole class situations (Rowell, Dawson & Lyndon, 1990). Thus the theoretical basis of O/N appears to relate equally well to misconceptions, alternative conceptions and inappropriate knowledge as it does to error patterns.

The Conceptual Mediation Program can easily fit within a constructivist framework of learning and acquiring knowledge. CM offers a strategy for accelerating the process of conceptual change, and aligns current trends in mathematics intervention research that acknowledges the potential of errors/misconceptions as starting points for intervention programs. CM offers a psychological perspective on the nature of errors/misconceptions, and thus provides a further dimension to the misconceptions jigsaw.

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